Exercise 29

Find $f'(x)$ and $f''(x)$.

$$f(x) = \frac{x^2}{1+e^x}$$

Solution

Use the quotient rule to differentiate $f(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{x^2}{1+e^x}\right) \\ &= \frac{\left[\frac{d}{dx}(x^2)\right](1+e^x) - \left[\frac{d}{dx}(1+e^x)\right](x^2)}{(1+e^x)^2} \\ &= \frac{(2x)(1+e^x) - (e^x)(x^2)}{(1+e^x)^2} \\ &= \frac{2x + 2xe^x - x^2e^x}{(1+e^x)^2} \end{aligned}$$

Use the quotient rule again to differentiate $f'(x)$.

$$
\begin{aligned}
f''(x) &= \frac{d}{dx}\left[\frac{2x+2xe^x-x^2e^x}{(1+e^x)^2}\right] \\
&= \frac{\left[\frac{d}{dx}(2x+2xe^x-x^2e^x)\right](1+e^x)^2-\left\{\frac{d}{dx}[(1+e^x)^2]\right\}(2x+2xe^x-x^2e^x)}{(1+e^x)^4} \\
&= \frac{\left\{2+\left[\frac{d}{dx}(2x)\right]e^x+(2x)\left[\frac{d}{dx}(e^x)\right]-\left[\frac{d}{dx}(x^2)\right]e^x-(x^2)\left[\frac{d}{dx}(e^x)\right]\right\}(1+e^x)^2-\left\{\left[\frac{d}{dx}(1+e^x)\right](1+e^x)+(1+e^x)\left[\frac{d}{dx}(1+e^x)\right]\right\}(2x+2xe^x-x^2e^x)}{(1+e^x)^4} \\
&= \frac{\left[2+(2)e^x+(2x)(e^x)-(2x)e^x-(x^2)(e^x)\right](1+e^x)^2-\left[(e^x)(1+e^x)+(1+e^x)(e^x)\right](2x+2xe^x-x^2e^x)}{(1+e^x)^4} \\
&= \frac{\left(2+2e^x-x^2e^x\right)(1+e^x)^2-(1+e^x)(2e^x)(2x+2xe^x-x^2e^x)}{(1+e^x)^4} \\
&= \frac{\left(2+2e^x-x^2e^x\right)(1+e^x)-(2e^x)(2x+2xe^x-x^2e^x)}{(1+e^x)^3} \\
&= \frac{2+4e^x+2e^{2x}-4xe^x-4xe^{2x}-x^2e^x+x^2e^{2x}}{(1+e^x)^3}
\end{aligned}
$$

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